Paradigm Shift

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June 10, 2011

Abstract

This paper analyses the consequences of young researchers' scientific choice on the dynamics of sciences. We develop a simple two state mean field game model to analyze the competition between two paradigms based on Kuhn's theory of scientific revolutions. At the beginning of their career, young researchers choose the paradigm in which they want to work according to social and personal motivations. Despite the possibility of multiple equilibria the model exhibits at least one stable solution in which both paradigms always coexist. The occurrence of shocks on the parameters may induce the shift from one dominant paradigm to the other. During this shift, researchers' choice is proved to have a great impact on the evolution of sciences.

Keywords : Paradigm shift, Scientific choice, Research dynamics, Mean field game.

JEL Classification : O39, C61.

1 Introduction

Social sciences, among other disciplines, are consistently subject to conceptual or methodological swings. In economics for instance, Transaction Costs analysis gave way to Agency Theory, Endogenous Growth appeared at the expense of standard Growth theory and, more recently, Behavioral Finance deeply challenged the standard Market Efficiency Hypothesis. Such an evolution suggests the existence of life cycles affecting research agendas or of paradigm shifts, a core concept developed in Kuhn (1970).

In a broad sense, a paradigm may be defined as a set of theories and empirical methodologies which allow a scientific community to identify frame and solve problems and serve as a foundation for future scientific discoveries. During periods of normal science, a paradigm allows reporting interesting or surprising findings and will remain dominant as long as it can stay attractive for the large majority of researchers. While this paradigm attracts the majority of scientists, two kind of academic fields are unlikely to attract a vast amount of researchers (Crane, 1969). They are either new topics for which the theoretical implications are not fully realized (pre normal science) or traditional research fields for which the immediate intellectual content has been exhausted and only remains difficult theoretical problems which are not really attractive for young researchers (post normal science). During a paradigm shift two simultaneous changes are supposed to occur: the decline of the old paradigm, when the paradigm begins to fail solving problems and explaining anomalies and the emergence of a new one if a new theoretical corpus allows the publication of promising results. During these changes, the hope of new discoveries modifies the researchers' scientific choices who progressively abandon the traditional research fields in favor of the new set of assumptions.

Driven first by scientific considerations, the paradigm shift also appears as a social fact involving the complete community of scientists. During crisis, the increase in the number of researchers involved in the new scientific approach induces a social phenomenon which will cumulatively foster its attractivity. More researchers in an academic field simultaneously increases the potential audience of a given research, rises the ease in finding efficient co-writers, guarantees an easier access to publication mediums and contributes simplifying the funding of research. The increase of the scientific community interested in a scientific field thus influences – per se – the researchers' scientific choice. When the new set of assumptions attracts most of a new generation of scientists, the older school disappears. Researchers who stay working in the old school see their influence decreasing and their contribution are rapidly marginalized.

Demographic elements also contribute to the dynamics of science. History of sciences gives various illustrations to the fact that the retirement of one generation of elite scientists and their replacement by a new generation allows the latter to develop more easily new theories or approaches (Barber, 1961).

Besides, one cannot neglect the stimulus brought to researchers through paradigm competition. According to Kuhn (1970), "*Competition between segments of the scientific community is the only historical process that ever actually results in the rejection of one previously accepted theory or in the adoption of another*". During periods of normal science, opponents to the dominant approach highlight the existence of anomalies which seem inconsistent with the leading paradigm. In answer, supporters of the paradigm spend a large part of their career in the process of puzzle solving, an activity which allows to comfort the established framework. Paradigm competition appears as one additional driving forces of scientific productivity.

This paper aims at considering the various determinants of the researchers' choice of their scientific issue and the consequences of these choices in the general evolution of science. If this approach clearly deals with various aspect of Khun's work, we do not claim to formalize his theory. Our purpose is to focus on the various conditions that could contribute to the decline of a paradigm and the shift to a new one. In this purpose, we build a highly stylized mean field game closely related to the description by Guéant (2009) of the workers' choices in a two sector economy.

In the paper at hand, we consider an economy with a continuum of researchers and two competing paradigms. Researchers produce homogeneous papers according to a production function which reflects both the development stage of the paradigm in which the scientists are involved and the repartition of the researchers among the two paradigms. At each point in time, a fraction of researchers quit academia and is replaced by an equivalent number of young researchers. Each of them has to choose in which paradigm he or she wants to carry out his or her work. Two factors motivate the choice of these young researchers at the beginning of their career: the intertemporal remuneration scheme (social or monetary) and the personal preferences.

A priori, the young researchers' scientific choice is firstly influenced by their affinity with the topic that they will handle for the rest of their life. They will choose between the various scientific topics according to their taste, given their attitude to risk, their greater or lesser reluctance to treat opened up questions or their desire to engage in riskier issues (Alon, 2009). However, in their choice, young scientists cannot ignore the influence of the remuneration scheme offered by each of the two paradigms. As any scientist, a young researcher seeks social recognition (Merton, 1957), a recognition which comes with the publication of new results and is dramatically linked to the possibility of creating and disseminating new knowledge (Stephan, 1996). Besides, monetary wages are also highly related to the academic resume and the individual scientific production (see for instance diamond swidler) In turn, as this scientific production is influenced by the proportion of researchers working in the same paradigm, the dynamics of the population distribution between the two paradigms has a crucial influence on the young researchers' choice.

According to the initial values of the parameters, the model exhibits one or two stable equilibriums. In each equilibrium, the two paradigms always coexist; one paradigm is dominant attracting the majority of the researchers while the other is dominated. In these equilibriums, coexistence is due to the voluntary choice by some young researchers of a research agenda in the dominated paradigm even if this agenda is not intended to lead to major innovation.

When the model allows for two stable equilibriums, the equations give no indication about which of the two competing paradigms should become predominant. Both paradigms could possibly become dominant and the hierarchy is inherited from history of the scientific field which drove to the initial repartition of the researchers among the two paradigms. In this case, a paradigm shift may occur if random shocks on some of the parameters value contribute to eliminate the dominant paradigm as a stable equilibrium. After such shocks, the vast majority of young researchers will be attracted by the new paradigm which allows for a rising remuneration as long as the number of researchers involved in the paradigm increases.

While analyses of the dynamics of sciences belong now to a well established field of research in economics, there are only few theoretical analyses that offer a formal model of paradigms evolution. As a related work, we can refer to Sterman and Wittenberg (1999) who provide a Kuhnian dynamic model in which paradigm changes are conditioned by positive feedback loops. Besancenot et al. (2011) worked out a hierarchical differential game between editors and authors. The production of scientific knowledge is analyzed as the extraction of potential knowledge from a paradigm seen as an exhaustible resource. Editors can accelerate or slow down knowledge production and paradigm depletion may occur when editors allow for a fast rate of knowledge extraction. In this model, paradigm depletion may be an optimal outcome. More recently Bramoullé and Saint Paul (2010) developed an overlapping generation model in which researchers allocate their working time between old or new fields of research in order to maximize the authors' reward. At each period, one paper published in a given paradigm yields both a citation premium increasing with the future number of contributions to the paradigm and a direct remuneration linked to the intrinsic value of the paper. The model exhibits solutions with various properties according to the values of the parameters. In some cases, the model allows for succession of periods of emergence of new paradigms and periods of exploitation of the old ones. In other cases, sunspots may occur where expectations of a high payoff in investment in a scientific field attract lots of researchers in the paradigm and allows for self-fulfilling expectations. Among the literature, our model presents a greater affinity with the work of Brock and Durlauf (1999) who developed a model in which researchers' scientific choice is made by reference to conformity. Their model puts a special emphasis on the tendency for individual scientists to place a greater weight on theories accepted by the majority of the academic community.[1] Under this assumption, the authors put forward a multiplicity of equilibriums and the possibility of jump from one equilibrium to the other in case of shock on the parameters. Our approach differs from this work in three ways. First we develop a model in which the arguments of the scientific choice are directly linked to the scientific reward scheme. In their choice, researchers perfectly take into account the future possibilities of papers production and the social and monetary rewards that come with the academic resume. Second, our model allows taking into account the demographic dimension of the problem and its influence on the paradigm shift. Third the model is built on the mean field game approach introduced by Lions and Lasry (Lasry and Lions, 2006a,b, 2007), see also Guéant (2009) for a thorough presentation. In order to formalize the behavior of a continuum of rational agents, the Mean Field Game Theory assumes each agent to be influenced by the mean field made of the distribution of other players' behavior and considers the consequences of each individual decision on this mean field[2]. In a standard Mean Field Game, the dynamics of

[1] Another topic close to this work deals with the occurrence of fads or cascades effects in sciences (Sunstein (1999), Starbuck (2009) and among others the paper of Abrahamson (2009) in the special issue of the Scandinavian Journal of Management dedicated to this subject.). Obviously, a fad may occur when people decide to do something just because other people are doing it. Fads effects are closely related to the conformity effect studied in Brock and Durlauf (1999).

[2] The Mean Field Game Theory adopts the methodological of statistical physic while modelizing the interaction of a great number of particles. Faced to this insurmountable computa-

the system is governed by two equations: a backward Hamilton-Jacobi-Bellman equation describing the optimal behavior of agents given the distribution of the other players and a forward Kolmogorov equation which takes into account the influence of each player on the mean field. The Nash equilibrium of the game appears as the solution of these two equations. In this paper, we consider a simplified model based upon a system of ordinary differential equations while keeping the same characteristics.

The paper is organized as follows. Section 2 introduces our main assumptions about the researcher's payoffs, their productivity and the dynamics of the model given the young researchers' choice. Section 3 presents the properties of the equilibriums. Section 4 provides a numerical simulation of the model for various values of the parameters and section 5 discusses the results in terms of paradigm shifts. A last section summarizes our conclusions

2 The model

We consider an academic world made up of a continuum of researchers of size 1. Each researcher practices his/her skills in one of the two available paradigms. Hereafter, a researcher working in paradigm i will be referred to as an $i - researcher$. Except for their preferences, researchers are assumed to be homogeneous.

At each point in time, a fraction λ of the researchers quit the academic world (through volunteer departure, or involuntarily through retirement or death) and is replaced by an equivalent number of young researchers. Young researchers have then to decide in which paradigm they want to carry out their research. This decision is definitive[3].

tional problem, physicists consider each particules as being influenced bt a "mean field"exerted by all other particles while simultaneously taking into account the influence of each particle on the mean field.

[3]The assumption that a young researcher makes a definitive choice of his/her problematic at the beginning of his/her career is purely technical. However, it perfectly matches with Khun's quotation of Max Plank : "a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it." Kuhn (1970), p.150. See also Morgenstern (1972) p. 1167 who writes that "the absorption of a new paradigm awaits, as a rule, a new generation" or (Barber, 1961) who give a careful description of the resistance by the senior researchers to scientific changes.

Basically, this choice depends on the researchers' reward structure which includes two different items: an intrinsic remuneration linked to the researcher's affinity with his research agenda, and an extrinsic one which results from his research activity.

2.1 Researchers extrinsic remuneration

The extrinsic reward of an academic work is composed of two different elements : a social remuneration linked to the interest paid by the scientific community to the researcher's work and a financial reward, typically the salary of the researcher. These elements will be formalized through three main variables:

- Let us denote by $Q_i(t)$ the number of papers published at date t by a representative $i-$researcher (papers quality is assumed to be homogeneous and $Q_i(t)$ also gives a qualitative measure of the scientific production of the $i - researchers$). According to Merton (1957), the scientific community awards recognition for being the first to communicate a new knowledge. Publication, which is a necessary step in establishing priorities, thus appears as a proof of efficiency and the larger the number of publications in an academic resume the higher the peer social recognition (Stephan, 1996). Moreover, the financial part of the researchers' reward is largely influenced by his/her academic resume. The role played by the number of publications or citations in an academic career has largely been documented in the academic literature (Hamermersh et al. (1982), Diamond (1986) or Swidler and Goldreyer (1998)). Hereafter, social and monetary rewards will thus be assumed increasing with $Q_i(t)$.

- Let $N_i(t)$ denote the number of $i - researchers$ at date t. Obviously, the greater the population of researchers potentially interested in a scientist's work, the more his / her work will be used or cited and the larger will be his scientific reputation. Thus, the researcher's social reward involved in the paradigm i is increasing with $N_i(t)$.

- Define by m_i the institutional component of the researchers' reward. As public or private funding agencies may want to promote one paradigm, they may offer special subsidies to researchers involved in this field of research. In the paper at hand, $m_i \in [1, \infty[$ measures the level of these monetary incentives. When $m_i = 1$, funding agencies provide no incentive for researchers to work in the scientific area i. For $m_i > 1$, the higher is m_i and the higher are the incentives to become a i-researcher.

Finally, the instantaneous value, $\omega_i(t)$, of the researchers' extrinsic remuneration (social and monetary) appears as a function of the previous variables:

$$\omega_i(t) = \omega_i(m_i, N_i(t), Q_i(t)),$$

and, at date t the intertemporal expected remuneration for an $i - researcher$ is given by:

$$u_i(t) = \mathbb{E}\left[\int_t^{t+T} \omega_i(m_i, N_i(s), Q_i(s))\, e^{-\alpha(s-t)} ds\right]$$

here T is a random variable that corresponds to the time spent in the research field i by an $i - researcher$ during his academic life. Assuming that this variable follows an exponential law of intensity λ, this last expression takes the simplified shape:

$$u_i(t) = \int_t^{\infty} \omega_i(m_i, N_i(s), Q_i(s))\, e^{-(\alpha+\lambda)(s-t)} ds. \tag{1}$$

2.2 Specific assumptions

In order to obtain tractable solutions, the $i - researcher$'s production function will be formalized through a classical CES function[4]:

$$Q_i(t) = \left(a_i N_i^r(t) + (1 - a_i)(N_{-i}(t) Q_{-i}(t))^r\right)^{1/r}. \tag{2}$$

where:

1. $N_i(t)$ is the number of i-researchers.
2. $N_{-i}(t)$ is the number of researchers in the competing paradigm. As the continuum of researchers is of size one, we have $N_{-i}(t) + N_i(t) = 1$.

[4] Under this assumption, the case $N_i = 0$ could rise a formal problem as the production function would allow for some scientific production in the field of research i while no researcher would be involved in this specific field. In our model, however, this difficulty is avoided as the case $N_i = 0$ is inconsistent with the equilibrium solution.

3. $N_{-i}(t)\,Q_{-i}(t)$ measures the number of papers published within the competing paradigm.

4. a_i is a specific constant measuring the dependence of paradigm i with respect to its rival. A high level of a_i reveals an autonomous field of research in which researchers are poorly influenced by the scientific activity of the other field.

The rationale behind such a function is straightforward. Other thing remaining the same, an i-researcher's productivity is fostered by the number $N_i(t)$ of researchers involved in the same paradigm. More researchers means more conferences in which one can receive critics about his work and discuss with other academic fellows the new scientific developments of he paradigm. More researchers involved in a scientific field also means more opportunity of collaborations which increase productivity (see for instance Mcdowell and Melvin (1983), Landry et al. (1996) or Abrahamson (2009)) and induces a greater number of reviews in which one can publish his/her work (Stigler et al., 1995).

Besides, competition between paradigms plays a crucial role on scientific productivity. During periods of normal science, while opponents to the dominant approach highlight the existence of anomalies which seem inconsistent with the leading paradigm, supporters of the paradigm spend a large part of their career to comfort the established framework. In economics, a good illustration of such a phenomenon can be found in the evolution of the efficient market hypothesis in reaction to the systematic research of anomalies in the financial market by supporters of behavioral finance (Schwert, 2003).

This opposition is formalized by the specific constant a_i which captures the intrinsic dynamism of the paradigm i and its stage in the paradigm shift. From its rise until its decline, a paradigm's life is subject to random shocks that affects its relation *vis–vis* its competitor. In the early years of the new paradigm i, some researchers are disappointed by the results of the dominant concepts and start pursuing alternative topics or methodology in the hope that a new set of tools or assumptions would bring better results. At this stage, the new approach defines itself by opposition to the dominant paradigm and a_i is rather low. However, a shock on a_i can occur if the new set of assumptions starts allowing to report interesting or surprising findings. In such a case, a_i increases as authors become more interested in the development of the new results than by the criticism of the old ones. Finally, a_i may decrease when the most important problems of the field are solved or proven to be unsolvable. In this case, new papers in the field bring fewer innovations and researchers will spend most of their time trying to answer the critics raised by the competing paradigm.

Finally, we assume that the instantaneous remuneration for a $i-researcher$ presents a multiplicative shape and is given by:

$$\omega_i(t) = m_i N_i(t) Q_i(t) \tag{3}$$

2.3 Intrinsic remuneration and the young researchers' choice

At the beginning of his academic life, each researcher has to choose the sector in which he/she will work for the rest of his/her life. In this choice, the remuneration offered by each field of research plays a determining role; however, the young researchers will also take into account their personal preferences among the various academic fields (Alon (2009), Stephan (1996)). In this paper, the researcher's preferences which induce his/her intrinsic remuneration are modeled by a random variable μ which measures the value for a young researcher of building his/her career in the first scientific area. By assumption each researcher is characterized by his/her own μ, and this value is distributed over the researchers' population according to a standard normal law.

When the two research agendas bring the same intertemporal remuneration $u_1(t) = u_2(t)$, Cf. Eq. (1), a researcher will choose the first paradigm for any μ positive and the second one for a negative μ. When the intertemporal remunerations exhibit significant differences, a young researcher may nevertheless choose the less remunerative if he/she exhibits strong preferences for this field of research. Formally, the decision rule for a young researcher will be to choose the first area if and only if[5]:

$$u_1(t) + \mu \geq u_2(t). \tag{4}$$

Let us reason on an infinitesimal interval $[t, t+dt]$. According to the previous assumptions, during this time period a proportion λdt of researchers retires both

[5]We made the assumption that the young researchers have perfect forseight.

for sector 1 and sector 2 and a population of size λdt enters the academic world. The proportion of new researchers that choose sector 1 is given by:

$$\mathbb{P}\left(u_1\left(t\right)+\mu\geq u_2\left(t\right)\right)=F\left(u_1\left(t\right)-u_2\left(t\right)\right), \tag{5}$$

where F is the cumulative distribution function of a standard normal variable.

Finally, the system is governed by the following two equations:

$$\begin{cases} \dot{N}_1\left(t\right)=-\lambda N_1\left(t\right)+\lambda F\left(u_1\left(t\right)-u_2\left(t\right)\right) \\ \dot{N}_2\left(t\right)=-\lambda N_2\left(t\right)+\lambda F\left(u_2\left(t\right)-u_1\left(t\right)\right) \end{cases} \tag{6}$$

Hereafter, we will use the variable $\Delta u=u_1-u_2$.

From (1) and (6), we can now describe the dynamics of the model:

Proposition 1 *The dynamics of the model is given by the two following equations*[6]*:*

$$\begin{cases} \dfrac{dN_1\left(t\right)}{dt}=-\lambda N_1\left(t\right)+\lambda F\left(\Delta u\left(t\right)\right) \\ \dfrac{d\Delta u\left(t\right)}{dt}=\left(\alpha+\lambda\right)\Delta u\left(t\right)-\left[\omega_1\left(N_1\left(t\right),N_2\left(t\right)\right)-\omega_2\left(N_1\left(t\right),N_2\left(t\right)\right)\right] \end{cases} \tag{7}$$

with an initial condition on N_1, $N_1\left(0\right)$, and a terminal condition on Δu, $\lim\limits_{t\to\infty}e^{-(\alpha+\lambda)t}\Delta u\left(t\right)=0$.

Proof. Remark that the first equation of the system (7) and (6) are formally equivalent. In a same way, (7) and the terminal condition verified by Δu are equivalent to the integral form (1) above. Indeed, after subtraction of the term $\left(\alpha+\lambda\right)\Delta u\left(t\right)$ from both sides of (7) and multiplication by $-e^{-(\alpha+\lambda)t}$ we get:

$$\frac{d\left[e^{-(\alpha+\lambda)t}\Delta u\left(t\right)\right]}{dt}=-e^{-(\alpha+\lambda)t}\left[\omega_1\left(N_1\left(t\right),N_2\left(t\right)\right)-\omega_2\left(N_1\left(t\right),N_2\left(t\right)\right)\right] \tag{8}$$

[6]The system of differential equations presented above is very typical of mean field game. The first equation which is forward can be identified to the Kolmogorov equation whereas the second one, backward, replaces the Hamilton-Jacobi-Bellman equation (Guéant, 2009).

After integration with respect to t, and under of the terminal condition, this is equivalent to:

$$-e^{-(\alpha+\lambda)t}\Delta u\left(t\right)=-\int_{t}^{\infty}e^{-(\alpha+\lambda)s}\left[\omega_{1}\left(N_{1}(t),N_{2}(t)\right)-\omega_{2}\left(N_{1}(t),N_{2}(t)\right)\right]ds, \tag{9}$$

which finally leads to Eq.(1):

$$\Delta u\left(t\right)=\int_{t}^{\infty}e^{-(\alpha+\lambda)(s-t)}\left[\omega_{1}\left(N_{1}\left(t\right),N_{2}\left(t\right)\right)-\omega_{2}\left(N_{1}\left(t\right),N_{2}\left(t\right)\right)\right]ds. \tag{10}$$

□

3 Properties of the stationary solutions

Proposition 2 *A stationary solution of the model is given by:*

$$N_{1}^{*}=F\left(\frac{1}{\alpha+\lambda}\left[\omega_{1}\left(N_{1}^{*},1-N_{1}^{*}\right)-\omega_{2}\left(N_{1}^{*},1-N_{1}^{*}\right)\right]\right) \tag{11}$$

Proof. . The stationary solution verify the following system :

$$\left\{\begin{array}{l}0=-\lambda N_{1}+\lambda F\left(\Delta u\right)\\0=\left(\alpha+\lambda\right)\Delta u-\left[\omega_{1}\left(N_{1},1-N_{1}\right)-\omega_{2}\left(N_{1},1-N_{1}\right)\right]\end{array}\right. \tag{12}$$

From these two equations we get :
$N_{1}^{*}=F\left(\frac{1}{\alpha+\lambda}\left(\omega_{1}\left(N_{1}^{*},1-N_{1}^{*}\right)-\omega_{2}\left(N_{1}^{*},1-N_{1}^{*}\right)\right)\right)$

The existence of such a solution is a simple application of the intermediate value theorem. Indeed, as $\omega_i=m_iN_iQ_i$, the difference $\omega_1-\omega_2$ is bounded, hence if we consider the function $f\left(N_1\right)=N_1-F\left(\frac{1}{\alpha+\lambda}\left(\omega_{1}\left(N_{1},1-N_{1}\right)-\omega_{2}\left(N_{1},1-N_{1}\right)\right)\right)$, we get $f\left(0\right)<0$ and $f\left(1\right)>0$. This gives the result. □

It now remains to study the dynamical properties of our system and the nature of each stationary solution. Let's consider the differential system without the terminal condition on Δu :

$$\left\{\begin{array}{l}\frac{dN_{1}(t)}{dt}=-\lambda N_{1}\left(t\right)+\lambda F\left(\Delta u\left(t\right)\right)\\\frac{d\Delta u(t)}{dt}=\left(\alpha+\lambda\right)\Delta u\left(t\right)-\left[\omega_{1}\left(N_{1}\left(t\right),1-N_{1}\left(t\right)\right)-\omega_{2}\left(N_{1}\left(t\right),1-N_{1}\left(t\right)\right)\right]\end{array}\right. \tag{13}$$

We linearize the system in the neighborhood of each stationary solution $(N_1^*, \Delta u^*)$.

$$\begin{cases} \frac{dN_1}{dt}(t) = -\lambda N_1(t) + \lambda \Delta u(t) F'(\Delta u^*) \\ \frac{d\Delta u}{dt}(t) = (\alpha + \lambda) \Delta u(t) - [\partial_1 \omega_1 - \partial_2 \omega_2 - \partial_1 \omega_2 + \partial_2 \omega_2] (N_1^*, 1 - N_1^*) N_1(t) \end{cases}$$

In order to determine the nature of the stationary solution, we have to study the eigenvalues of the following matrix :

$$M = \begin{pmatrix} -\lambda & \lambda F'(\Delta u^*) \\ -[\partial_1 \omega_1 - \partial_2 \omega_2 - \partial_1 \omega_2 + \partial_2 \omega_2] (N_1^*, 1 - N_1^*) & \alpha + \lambda \end{pmatrix}$$

Proposition 3 *The only trajectory compatible with the terminal condition on Δu is the trajectory that converges towards the saddle point.*

Proof. Imagine that the terminal condition is verified on a trajectory that diverges. Since ω_i is bounded there exists $C > 0$ such that $\forall N \in [0, 1]$, $|\omega_1(N, 1 - N) - \omega_2(N, 1 - N)| \leq C$, hence

$$\begin{aligned} \Delta u(t) &= \int_t^\infty \omega_1(N_1(t), 1 - N_1(t)) - \omega_2(N_1(t), 1 - N_1(t)) e^{-(\alpha+\lambda)(s-t)} ds \\ &\Rightarrow |\Delta u(t)| \leq \int_t^\infty |\omega_1(N_1(t), 1 - N_1(t)) - \omega_2(N_1(t), 1 - N_1(t))| e^{-(\alpha+\lambda)(s-t)} ds \\ &\Rightarrow |\Delta u(t)| \leq C \int_t^\infty e^{-(\alpha+\lambda)(s-t)} ds \\ &\Rightarrow |\Delta u(t)| \leq \frac{C}{\alpha + \lambda} \end{aligned}$$

But, by assumption, $\lim_{t\to\infty} |\Delta u(t)| = +\infty$. This is absurd then this trajectory is not compatible with the terminal condition on Δu. $\square$

4 Numerical simulations

We have seen above that the differential system admits stationary solutions but the number of these solutions depends upon the value of the variables a_1, a_2, m_1 and m_2. In this section, we consider two important cases presented in table 1 . We are going first to study in each case the stationary system and then turn to the nature of these stationary solutions. Hereafter, we will take $r = 0.25$. We recall that the stationary solutions are given by:

$$\begin{cases} N_1{}^* = F\left(\dfrac{\omega_1\left(N_1^*, 1-N_1^*\right) - \omega_2\left(N_1^*, 1-N_1^*\right)}{\alpha+\lambda}\right) \\ \Delta u^* = \dfrac{\omega_1\left(N_1^*, 1-N_1^*\right) - \omega_2\left(N_1^*, 1-N_1^*\right)}{\alpha+\lambda} \end{cases}$$

case	a_1	a_2	m_1	m_2	Number of stationary solutions
Case 1	0.5	0.5	1	1	3
Case 2	0.2	0.6	1	1	1
Case 3	0.6	0.2	1	1	1
			1	1.5	3

Table 1: Table of parameter value

4.1 Case 1

See Table 1 for the data of the problem. We first resolve the fixed point problem of Proposition 2. In Figure 1 we plot the graph of the identity function on $[0, 1]$ and the function $N_1 \mapsto F\left(\dfrac{\omega_1 - \omega_2}{\alpha+\lambda}\right)$. It shows the existence of three fixed point which are $N_1^{*,1} = 0.0493$, $N_1^{*,2} = 0.5$ and $N_1^{*,3} = 0.9506$.

To study the nature of each stationary solutions, we have to compute the determinant of the matrix M in each stationary solution. The result is summarized in table 2.

Fixed point	value	Determinant	Nature
$N_1^{*,1}$	0.0493	-0.0023	Saddle point
$N_1^{*,2}$	0.5	0.0020	Repulsive point
$N_1^{*,3}$	0.9506	-0.0023	Saddle point

Table 2: Dynamical properties of stationary solutions

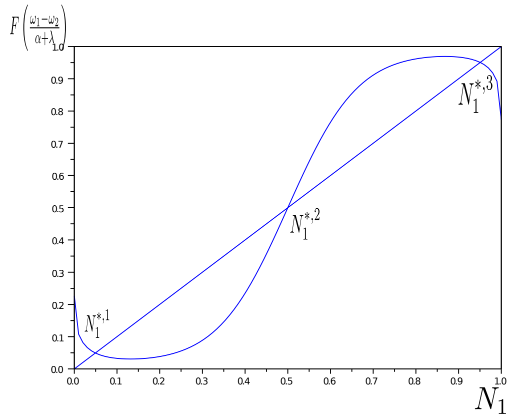

Figure 1: Case 1

4.2 Case 2

See Table 1 for the data of the problem. In this case $a_1 < a_2$. The unique fixed point is equal to $N_1^* = 0.0083$ as shown in figure. The stationary solution of the system is a saddle point since the determinant of the matrix M is equal to -0.0047.

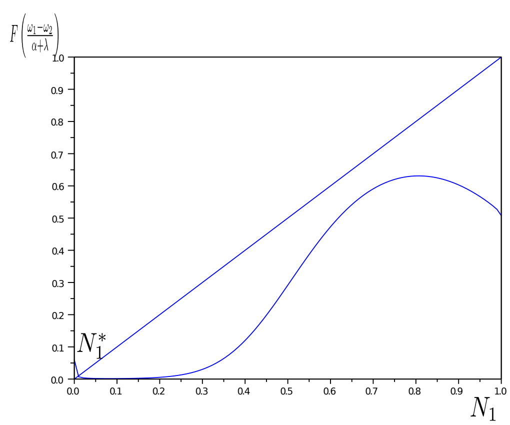

Figure 2: Case 2

Fixed point	value	Determinant	Nature
$N_1^{*,1}$	0.2461	-0.003	Saddle point
$N_1^{*,2}$	0.5024	0.0034	Repulsive point
$N_1^{*,3}$	0.9916	-0.0046	Saddle point

Table 3: Dynamical properties of stationary solutions

4.3 Case 3

See Table 1 for the data of the problem. In this example $a_1 > a_2$. When $m_1 = m_2 = 1$, the system admits a unique fixed point equal to $N_1^{*,3} = 0.9916$. Hence the unique stationary solution of the system is $N_1^{*,3} = 0.9916$ and $\Delta u^* = 2.3953$. This solution is a saddle point since the determinant of the matrix M is equal to -0.0047 and it leads to a domination of the first paradigm. The institutional factor may change considerably the dynamic properties of the system as shown in figure 3. When $m_2 = 1.5$, there exist three stationary solutions as represented by the dashed curve. The results are summarized in table 3.

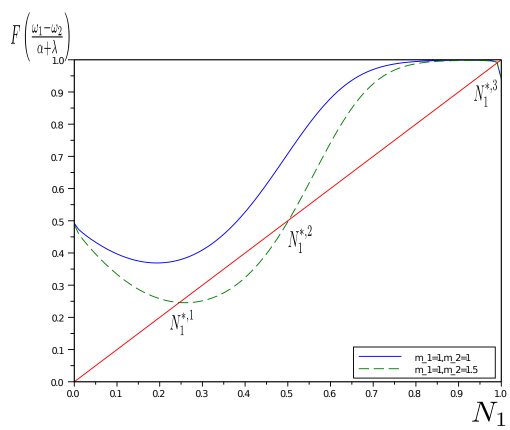

Figure 3: Case 3

5 The paradigm shift

Whatever the case considered in the previous section, the two competing paradigm always coexist. However, in the stable equilibriums the academic landscape is asymmetric by nature. One scientific approach appears as dominant, attracting a large majority of the researchers, while the other one, clearly dominated, is minority. Coexistence is guaranteed in this equilibrium because each paradigm is the complement of the other. The dominant field of research is stimulated by the researchers' critics from the competing research field while these researchers find easily matter of criticism in the massive scientific production of the dominant paradigm. Note that, in case 1, the two paradigms could potentially be dominant. The hierarchy between the two paradigms is then only due to historical choices of past researchers who mostly chose one of the two paradigms. As the equilibrium is stable, this paradigm may stay dominant for long.

In this model, paradigm shifts may appear as the consequence of successive and unanticipated shocks on the relatives values of a_i.[7]

During periods of normal science, researchers only focus on the development of the dominant paradigm (hereafter paradigm 2). Results are considered as significant when they add to the scope and the precision with which the paradigm can be applied and, in these settings, puzzle solving is the best way to increase the generality of the paradigm. Researchers are mainly interested in the improvement of the assumptions, the procedures or of the generality of the results inside the paradigm; a_2 is close to one. In such a period, results from the dominated field of research are neglected in the scientific debate and researchers involved in these topics have to define themselves by opposition to the dominant paradigm. In this case, with a high a_2 and a low a_1, the equilibrium is described by figure 2.

Apparition of anomalies brings an important shock to the model and changes the nature of the equilibrium. As more and more puzzles appear inconsistent with the dominant concepts, young researchers start pursuing alternative topics or methodology in the hope that a new set of tools or assumptions would bring better results. New possibilities of analysis are considered by these authors who start studying these problems with a greater autonomy, a_1 rises. In the same time, researchers from the dominant paradigm have to spend more time to address the critics of their challengers: a_2 drops. Under our specific assumptions this implies a lower number of publications in the dominant field and a slide in

[7]Kuhn (1970) provides various illustrations that the decisive shocks that will affect a paradigm are hardly expected by scientists. See for instance Pauli's pessimistic correspondence about the future of physics at the very period which gives birth to Quanta's theory (Kuhn, 1970) pp. 83-84.

the social and monetary remuneration for researchers involved in this paradigm. In the other field of research, the opposite effects are at work and this field becomes more attractive for young researchers. When a radical change affects a_1 and a_2, the model can reach the situation described by the continuous curve of Case 3. In this situation, the model presents a unique stable equilibrium in which the old paradigm leaves its place to a new dominant one. The paradigm shift occurs according to an adaptation process from the old equilibrium to the new one. During this shift, the number of researchers attracted by the new topics raises continuously as young researchers is attracted in a cumulative way by the new remuneration schemes. Greater social recognition and higher wages are both the incentives that attract the young scientists in the new paradigm and the consequence of this massive attraction. At the end of the adjustment process, a new steady state is reached in which proponents of the old paradigm remain active - but with a minority status.

Note that the paradigm shift may also be caused or hindered by public policy. Indeed, goverment may choose to consolidate the dominance of one paradigm, for instance paradigm2, by funds and other types of support (interpreted here by an increase of the institutional factor m_2). This policy can be seen as justified in period of normal science and may last as long as the dominant paradigm presents no failure, but if new puzzles appear and empirical anomalies challenged its theoretical implications then policymakers face two choices : ignoring these anomalies and maintaining the same policy, described by the dashed curve of figure 3 where the equilibrium is N_1^* or abandoning the dominant paradigm and stop supporting it which lead to a paradigm shift and a new equilibrium as described by the continuous curve of figure 3.

6 conclusion

The two state mean field games developed in this paper models the competition between two paradigms in an academic field. The model accords a central role to the young researchers' choice in the dynamic of science. Researchers have perfect foresight and choose their scientific field according to their own tastes and given the intertemporal rewards provided by the two competing paradigms.

Three major insights emerge from the model. First, for any set of parameters, there always exists stable steady state equilibrium. In this equilibrium, both paradigms coexist in a hierarchical order. Second, changes in the reward schemes are able to challenge this hierarchical order. An increase in the productivity in one paradigm or the implementation of incentives in favor of one of the two paradigms clearly contributes to the reinforcement of this set of assumptions and tools. Moreover, when young researchers observe a decline in the activity within the dominant paradigm, they anticipate the end of the period of normal science which motivates them to look for others concepts or methods to treat the anomalies. Third, for important shocks on the parameters, the equilibrium with the dominating paradigm may disappear. In this case, one can observe a paradigm shift with the progressive replacement of former major scientists involved in the old paradigm by new generations of researchers, an increasing share of which choosing the new paradigm.

In our model, a paradigm shift appears as the consequence of two kind of unpredictable chocks. A chock in the scientific production functions may modify the interaction between the paradigms And favor one paradigm at the expense of the other. If a research agenda allows for an increase in the productivity of its researchers, the associated private and social rewards will rise and its scientific attractiveness will be enhanced. The institutional factor may also play a dramatic role in the choice of junior researchers and consequently in establishing a new hierarchy between paradigms. A government can foster the paradigm shift by providing temporary incentives to young researchers choosing the publicly encouraged set of assumptions. If these incentives reach a given threshold, the former equilibrium may disappear and the convergence to the new paradigm will be self-sustaining. After some adjustment in the relative number of researchers involved in the two paradigms, the incentive may be removed without questioning the paradigm shift.

In order to keep the analysis tractable, this paper is built on some restrictive assumptions. For instance, the model considers that young researchers make a definitive choice at the beginning of their academic life; future work should consider the possibility of a radical revision of a researcher's research agenda. Moreover, in order to obtain an analytical characterization of the equilibrium solution, some specific assumptions have been made about the reward structure and the functional forms of the academic production. These assumptions may be questioned in order to assess the accuracy of the model. Despite its limitations, the model is interesting as it allows stressing the role of the reward scheme in the dynamic of science and gives an overview of possible applications of mean field games.

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